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A Case Study of Network Design for Middle East Water Distribution

Submitted in partial fulfillment of the requirements for the degree of Master of Science in
Operations Research at Virginia Commonwealth University

by

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Abstract

The Middle Eastern region encompassing Israel, Jordan, and the Palestinian Territories (West Bank and Gaza) is an arid region with fast growing populations. Adequate and equitable access to water for all the people of the region is crucial to the future of Middle East peace. However, the current water distribution system not only fails to provide an adequate and equitable allocation of water, but also results adverse impacts on the environment. This project involves building a mathematical model to aid decision-makers in designing an optimal water distribution network. A new method for incorporating uncertainty in optimization that is based on Bayesian simulation of posterior predictive distributions is used to represent uncertainty in demands and costs. The output of the model is a most-probable least-cost modification to the existing water distribution infrastructure. Additionally, the model output includes the probability that a network component (new desalination plant, new pipe, new canal) is part of a least-cost installation.

Chapter 1: Introduction

The Problem

Water scarcity is a world-wide problem that has implications for public health, food supply, ecosystem health, and political stability. The problem of water scarcity, if not addressed, will get worse in the future. Areas of the world that are currently facing water deficits may be further stressed as demand for water increases with population growth and a rising standard of living. Additionally, existing supplies of freshwater could be threatened by shifting weather patterns as a result of climate change.

The Middle Eastern region encompassing Israel, Jordan, and the Palestinian Territories (West Bank and Gaza) is especially vulnerable to water stress as it is one of the most arid regions on earth and it has one of the fastest growing populations. Adequate and equitable access to water for all the people of the region is crucial to the future of Middle East peace. However, the current water supply and transmission system is failing to satisfy demand or provide an equitable allocation of water. Furthermore, the continued overexploitation of water resources has resulted in a decline in water quality and in adverse impacts on the environment.

This project involves building an optimization model that allows decision makers to evaluate various alternatives for a Middle East water supply and transmission network plan

that takes that will satisfy the population's forecasted demand in 2025 in a cost-effective, equitable manner.

Background

Israel derives one third of its fresh, natural water from Lake Kinneret (the Sea of Galilee) basin and the remainder from the streams and groundwater in the Mountain Aquifer and the Coastal Aquifer. Beginning in 1960s, Israel undertook an ambitious program to build the National Water Carrier (NWC) to transport water from the sources in the northern part of the country to the drier southern agricultural region. At its inception, 80% of the natural fresh water was supplied by the NWC went to agriculture, but the situation now is completely reversed, with urban areas receiving 80% of the water for drinking water and for domestic purposes. As the municipal demand for water grows with population growth and a rising standard of living, the allocations of fresh water to agriculture have decreased dramatically. Even with the curtailment in agricultural demand (made possible, in part by the substitution of brackish and treated wastewater), total demand for water remains greater than the available natural resource. A drought from 1999 - 2003 and again the last two years has exacerbated the problem. The water level of Lake Kinnerett, the main source of Israel's surface water, is monitored daily and compared to three safety lines, an upper and lower "red" line, and the "black line." If the water level of the lake falls below the red line, there may be damage to the water source, and withdrawals must be slowed. If the water level falls below the black line, "irreparable damage" is done and it is no longer physically possible to pump water from the lake into the NWC. Since the fall of 2009, the level of Lake Kinneret has been below the lower red level, and according to the projections of The Government Authority for Water and Sewage, "in the next 5 years it will not be possible to reach a level

above the red lines for all of Israel's water reservoirs" (Fixler, 2010).

Like many of the countries in the region, Jordan suffers from a lack of rainfall — 80% of the country received less than 100 mm of precipitation per year, and of that, 93.9% evaporates before it can recharge the groundwater. This is unfortunate, as Jordan must derive most of its water from groundwater resources since the Jordan River and its tributaries, the Zarqa and Yarmouk, are undependable due to over-pumping by Israel and Syria, and pollution from the Amman-Zarqa area (Mohsen, 2007). The water stress has become worse in Jordan over the last two decades as tens of thousands of Iraqi refugees fled to Jordan during the first and second Gulf Wars, causing a spike in population and a subsequent increase in water demand.

The Palestinians rely almost entirely on ground water from wells in the West Bank (Mountain Aquifer) and in the Gaza strip (Coastal Aquifer). All of the water wells in the Gaza Strip are under Palestinian control, but that is not the case in the West Bank where Mekorot, the Israeli national water company, owns a large number of the wells installed for the settlements and kibbutz. In addition to the water obtained from their own well, the West Bank Palestinians purchase water from Mekorot, some of which is supplied from outside of the West Bank.

If the population of the region grows as expected, and if the per capita demand for water increases as the economic status of the people of the region improves, future water deficits will only increase. Water deficits in the greater Middle East region are expected to reach 2,250 MCM by 2040 (Qdais, 2008), and the negative impacts of this shortfall will be particularly strong in Israel, Jordan, and Palestine. Already, the residents of the area are some of the most water-impooverished people in the world. In many neighborhoods in Amman, for example, water may only be available once a week, straining the ability of residents to meet their entire domestic needs (Lipchin, 2007). Additionally, there is a disparity in the amount of water

received on a per capita rate between the nations, with Israelis consuming approximately 3.5 times the amount of domestic (household) water as is available on a per capita basis to the Palestinians. This situation aggravates the resentment the Palestinians have for Israeli control and hampers the peace process in the region.

Political conflict is not the only problem that results from the water shortage in the region; the natural environment also suffers. Over withdrawals from aquifers allow salt water and other pollutants to intrude, resulting in a decline in water quality, and over withdrawals from surface waters deprive natural habitats (and the plants and animals they support) of the water they need. One of the most notable environmental impacts of the current water management system has been the decline of the Dead Sea. The water projects of Israel and Jordan divert almost all of the water that would naturally flow from the Jordan River into the Dead Sea, causing the water levels of the Dead Sea to decline between 0.5 and 0.95 m/year (Asmar, 2003) and the surface area of the lake to shrink by 33% in the last 55 years. The declining water level has resulted in sinkholes forming along the shoreline, destruction of roads and bridges, ecological disruption, and the loss of water from the surrounding freshwater aquifers. This in turns threatens the important tourism business, the local potash industry, several endangered species, human health, and regional stability (Qdais, 2008).

Potential Solutions

Decision makers in the Middle East are currently studying various alternatives to address water scarcity and the Dead Sea restoration. Potential projects include a series of desalination plants along the Mediterranean, a water connection between the Mediterranean Sea and the Dead Sea (here after called the Med-Dead Water Conveyance or M-DWC), a water connection between the Red Sea and the Dead Sea (Here after called the Red-Dead Water

Conveyance or R-DWC), or a combination of such projects. The World Bank is currently financing a study of the Red Sea — Dead Sea Water Conveyance Project, at a cost of \$15.5 million dollars, indicating the level of importance placed upon reaching a solution that will meet current and future water demands in a sustainable manner (The World Bank, 2007).

Israel's Desalination Master Plan

To address both the severe water shortages projected for the future and the continued deterioration in groundwater quality, in 1996 the Israeli government initiated studies to form the basis of a Desalination Master Plan. The plan projected water demands over a 20 year period, identified potential desalination plan sites along the Mediterranean coast, and examined the costs of producing desalinated water and delivering it to the national water supply system (Dreizin et al., 2008). At that time, six locations were identified in the plan as the most suitable for desalination. However, soon after the Desalination Master Plan was completed, a prolonged drought led to a national water crisis and a need to build desalination plants as quickly as possible. As of early 2010, plants have been built at two of the locations identified in the Master Plan, Ashkelon and Hadera, and a third plant has been built at Palmachim.

Research Objectives

This project involves building a mathematical model to aid decision-makers in designing an optimal water distribution network. Chapter 2 discusses modeling and solving the problem for competing objectives: minimizing cost and maximizing per capita water supply equity. Chapter 3 discusses a new method to incorporate uncertainty in the model inputs. In both chapters, the objective is to decide which potential desalination plants to add to the

existing water distribution system, which potential connections between desalination plants and demand centers to add to the system, and how much water should be produced at each plant and flow along each connection in order to satisfy the demand at each demand center at the least total cost (cost of building new infrastructure and operating the new and existing infrastructure).

Chapter 2: Finding an Equitable Solution

Math Programming Model of the Middle East Water Distribution Network

The Middle East water distribution problem can be modeled a network optimization problem as it naturally lends itself to the definition of a network:

- Network — a set of nodes and a set of arcs
- Nodes — set of points connected by arcs to form a network
- Arc — an ordered pair of nodes that represents a possible direction of motion between nodes.
- Capacity — the quantity of a product that can pass through a given arc
- Cost — the financial expense for transporting a product through a given arc
- Source (S) — starting point in a network problem
- Sink (T) — terminal point in a network problem

Figure 1 is a graphical representation of a network as described by:

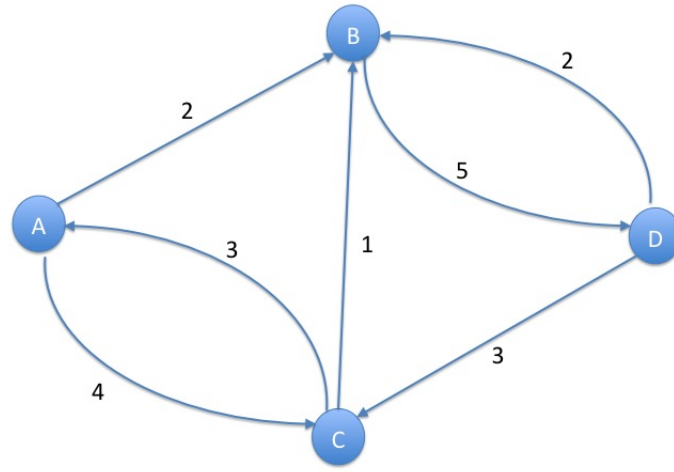


Figure 1: Example of a network

Nodes: {A, B, C, D}
 Arcs: {(A,B), (A,C), (B,D), (C,A), (C,B), (D,B), (D,C)}
 Capacity: {(A,B)=2, (A,C)=4, (B,D)=5, (C,A)=3, (C,B)=1, (D,B)=2, (D,C)=3}
 Sources: A, D
 Sinks: B, C

Middle East Water Distribution Network

In the model of the Middle East water distribution system as a network the source nodes represent the desalination plants, lakes, and groundwater sources; the demand nodes represent the cities (or population centers); and the arcs represent the canals, pipelines, and tunnels that connect the sources to the sinks. The existing water distribution system in Israel, the National Water Carrier (NWC), delivers water from Lake Kinneret (the Sea of Galilee) in the north to the coastal cities and eventually to the agricultural region in southern Israel (the Negev). The NWC consists of a system of underground pipes, open canals, interim reservoirs and tunnels, which is simplified in the network representation as 13 intermediate nodes (they are neither source nodes nor sink nodes). Included in the network are three



Figure 2: Existing water distribution network. This map background and the background of all maps in this document are courtesy of UNEP/DEWA/GRID-Europe (United Nations Environmental Program, 2010)

supply points (source nodes) of fresh, natural water: Lake Kinneret, the Mountain Aquifer, and the Coastal Aquifer, and five desalination plants that add supply to the NWC: (from north to south) Hadera, Palmachim, Granot, Gat, and Ashkelon. Additionally, there are desalination plants that are currently not connected to the NWC but supply water to local demand such as the three Sabah plants that supply Eilat. The network considers thirteen demand points (sink nodes) across Israel as well as the demand for fresh water for agriculture as represented by one demand node at the southern end of the NWC. The Palestinian

demand is represented by one demand point for the Gaza Strip and four demand points in the West Bank. The West Bank does not currently have a comprehensive water distribution network, but rather regional piping networks connected to Palestinian and Israeli (Merkrot) wells as all the fresh water available in the West Bank is groundwater (Palestinian National Authority and Palestinian Water Authority). Additionally, some water is supplied from the Israeli water system to the West Bank. The existing Jordanian water distribution system mostly moves water from the Jordan River up to Amman and the other population centers in the NW part of the country, but is not fully connected as in the system in Israel. In this model, Jordanian demand is accounted for by three demand points centered in the areas of high population: Amman, Az Zarqa, and Irbid. Using the locations identified in the Israeli Desalination Master Plan and recently reported proposals (Global Water Intelligence, 2009) potential desalination plants along the Mediterranean could be located at Sorek, Palmachim, Ashdod, Shomrat, Haifa, Rishon Le Zion, Netanya, and Tel Aviv. Finally, the large water conveyance projects such as the M-DWC (Mediterranean pipe/canal to western Dead Sea with desalination) or R-DWC (Red Sea pipe/canal to southern Dead Sea with desalination) could be built to supply water to the region as a whole and restore the Dead Sea. A graphical representation of the network problem with existing supply nodes and arcs is shown in Figure 2; potential nodes and arcs is show in Figure 3.

A problem modeled as a network can then be solved for different objectives. For example the shortest path (in cost or distance) between two nodes, the maximum flow of the network, and the minimum cost network flow (the minimum cost way to satisfy the demand at all the sinks).

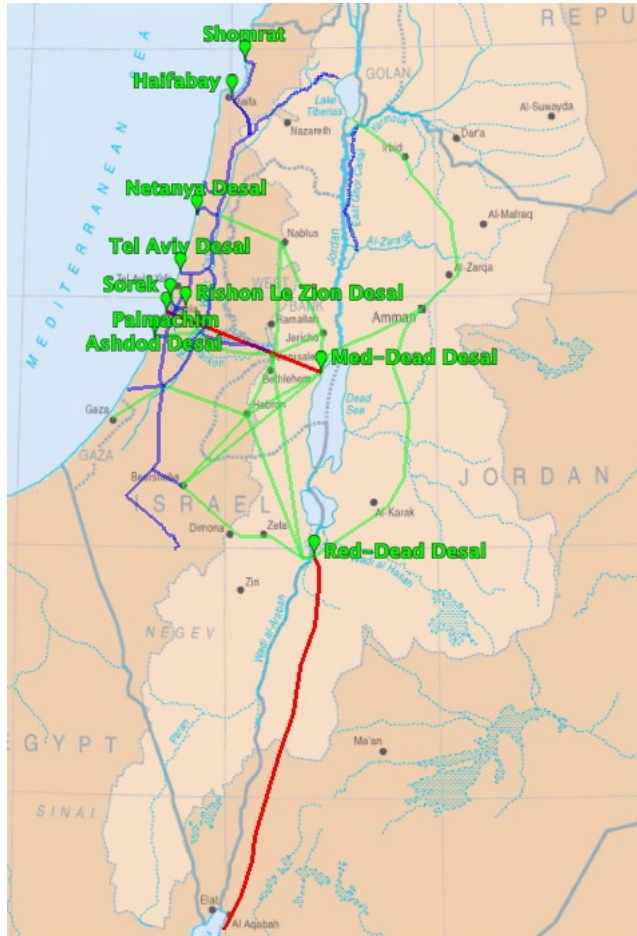


Figure 3: Potential water distribution network. Existing network elements are blue, potential new network elements are green, and the proposed large water conveyance projects are shown in red.

Multiple Objectives in the Middle East Water Distribution Network

Finding an “optimal” water distribution network in the Middle East is complicated because of the presence of multiple objectives. Most obvious is the objective of satisfying current and future demand for water at the least cost. However, seeing that water is intrinsically linked to political stability in the region, finding a solution that maximizes cooperation among the Middle Eastern states can also be considered optimal. The notion of what makes the water

distribution network an “optimal” network may also be different among the Middle Eastern states. Based on past actions and statements by the governments in the region, maintaining sovereign control over the source of water is highly important (which may be at odds with the idea of interstate cooperation and control of water sources). In the case of the Palestinians and Jordanians, increasing their citizen’s per-capita share of water is seen as critical for the economic development of their nations, therefore minimizing the per-capita disparities in water availability between the Israelis and the Palestinians and Jordanians may be another way to optimize the network. Finally, there are the supporters of the natural environment in general and the Dead Sea in particular who would like to see a water distribution network that takes into account “nature’s” needs. In this case an optimal water distribution network could be one that minimizes negative environmental impacts, or minimizes the further decline in the Dead Sea water level, or instead maximizes the restoration of the Dead Sea.

This chapter presents a method to solve the Middle East water distribution network problem for two different objectives: minimum cost and maximum equity. It is assumed that both these objectives can not be optimized at the same time, so intermediate solutions are found that strike a balance between cost and equity.

To minimize the cost of the Middle East water distribution network the problem is modeled as a fixed-charged network flow (FCNF) problem with the following objective: decide which potential desalination plants to add to the existing water distribution system, which potential connections between desalination plants and demand centers to add to the system, and how much water should be produced at each plant and flow along each connection in order to satisfy the demand at each demand center at the least total cost (cost of building new infrastructure and operating the new and existing infrastructure). The addition of the decision variables changes the formulation of this problem from a linear program to a mixed integer program (MIP).

Fixed Charge Network Flow Model

For the model we specify a network $G = (N, A)$ whose nodes N are comprised of the largest cities in the region (demand points), and surface water sources and desalination plants (supply points). The set of arcs A includes the actual and potential connections (pipes, canals, etc.) from the supply points to demand cities and the actual and potential connections between pairs of cities. The planning objective is to minimize the total installation and operational costs of the water distribution network and desalination plants. Capital costs include purchasing land, building infrastructure, etc. Operating costs include supplies, chemicals, labor, energy, etc.

The Decision Variables

The decisions involved in installation and operation include:

- whether to install a new desalination plant and/or expand capacity at an existing plant,
- whether to install a new water conduit and/or increase the capacity of an existing conduit, and
- how much water should flow between locations.

The variables in the problem are listed in Table 1.

Table 1: Variables

| Notation | Description of Variables |
|----------|--|
| f_{ij} | Continuous; the flow along arc (i, j) |
| u_{ij} | Continuous; the additional capacity needed along arc (i, j) |
| x_i | Binary; indicates if additional supply is installed at node i |
| y_i | Continuous; the percentage of the supply capacity to be used at node i |

The model parameters, the cost and demand parameters, are shown in Table 2.

Table 2: Model Parameters

| Notation | Description of Parameters |
|------------|--|
| b_i | Demand at each demand node (MCM/year) |
| b_i^+ | Current + potential supply at each supply node (MCM/year) |
| U_{ij} | Current pipe capacity (MCM/year) |
| h_i^b | Annualized capital cost of installing max capacity at supply node (\$) |
| c_i^b | Annual operating costs of supply nodes: (\$/MCM/year) |
| h_{ij}^U | Annualized capital cost of adding capacity to pipelines/ canal (\$/MCM/year) |
| h_{ij}^U | Annual operating cost of pipelines/ canals (\$/MCM/year) |

The objective is expressed in the optimization model as follows:

$$\min \sum_{(i,j) \in A} (c_{ij}^U f_{ij} + h_{ij}^U u_{ij}) + \sum_{i \in N} (c_i^b y_i + h_i^b x_i), \quad (1)$$

subject to

$$\sum_{j:(i,j) \in A} f_{ij} - \sum_{j:(j,i) \in A} f_{ji} = b_i + b_i^+ y_i \quad i \in N \quad (2)$$

$$f_{ij} \leq U_{ij} + U_{ij}^+ u_{ij} \quad (i, j) \in A \quad (3)$$

$$y_i \leq x_i \quad i \in N \quad (4)$$

$$u_{ij} \in \{0, 1\}, (i, j) \in A; x_i \in \{0, 1\}, i \in N; 0 \leq y_i \leq 1, i \in N$$

The first term of the objective function corresponds to the cost of the water transmission pipes and canals, where the parameter c_{ij}^U includes the per-unit operating, maintenance, pumping, and environmental costs and depends on and the amount of flow between nodes i and j (f_{ij}). The parameter h_{ij}^U includes the annualized capital costs of building additional capacity and depends on how much new flow capacity (u_{ij}) has to be installed between nodes i and j to accommodate the needed flow.

The second term of the objective function corresponds to the cost of the water supply nodes, be they desalination plants or surface and ground water supply points. The parameter

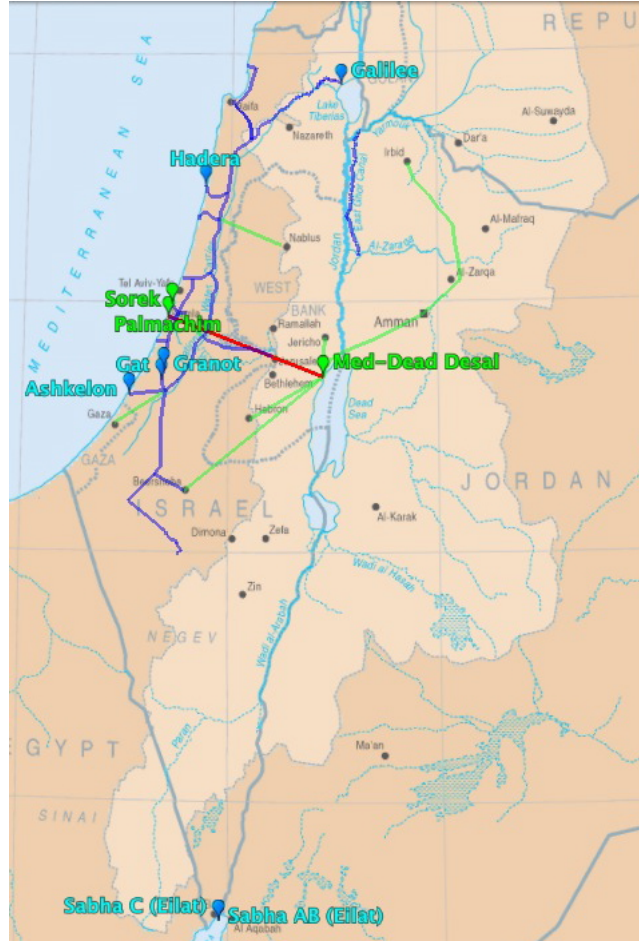


Figure 4: Minimum cost solution with unequal per-capita demand. The existing network elements are blue, the network elements installed in this solution are green, and the Med-Dead Water Conveyance is shown in red.

c_i^b includes the annual operating, maintenance, and environmental costs and depends on the amount of water supplied from node i as expressed as the percentage of the nodes maximum capacity (existing capacity + new capacity) (y_i). The parameter h_i^b includes the annualized capital costs of building a new desalination plant and depends on whether or not a new plant is built at node i (x_i). Existing desalination or surface supply points are dealt with in the model by setting their installation costs (annualized capital costs) to zero.

The first constraint, equation 2, enforces flow balance - the flow out of a node minus the flow in is equal to the supply or demand at the node; the second constraint, equation 3, enforces the capacity constraints on the arcs, and the third constraint, equation 4, sets the variable relationship between x and y .

The water demand parameters (b_i) are calculated by multiplying the current per-capita water use rate for each country by the projected population at each node in the year 2025. However, the current per-capita domestic water use rate is not equal among the states, with Israelis using on average $104 m^3$, Jordanians using $60 m^3$, and Palestinians using $34 m^3$ per-person per-year. It is assumed that if the Jordanians and Palestinians has access to more water, they would use more water. A more equitable situation would be to design a water distribution network that would provide for all people to have almost equal (on a per-capita basis) amount of water available. Therefore a second set of water demand parameters is calculated by multiplying one per-capita use rate ($100 m^3$) times the projected populations at each node.

The model is solved for both sets of demand parameters and two solutions with different objective function values are obtained. The first solution has a lower objective function value (lower cost) as it provides just enough water to each city to meet the population growth, but not enough to the Jordanian and Palestinian cities to increase their per-capita water availability equal to Israel (lower equity). This solution, seen in Figure 4, involves building new desalination plants at Sorek and Palmachim and constructing the Med-Dead Water Conveyance for a total cost of \$1.03B. Jordanian demand is met by the new Dead Sea desalination plant, which also supplies Jericho, Hebron, and Beesheba. Israeli demand and demand in Gaza and the Northern West Bank (Nablus) is met by adding the supply from the Mediterranean desalination plants to the existing Israeli National Water Carrier.

The second solution has a higher objective function value (higher cost) as it provides

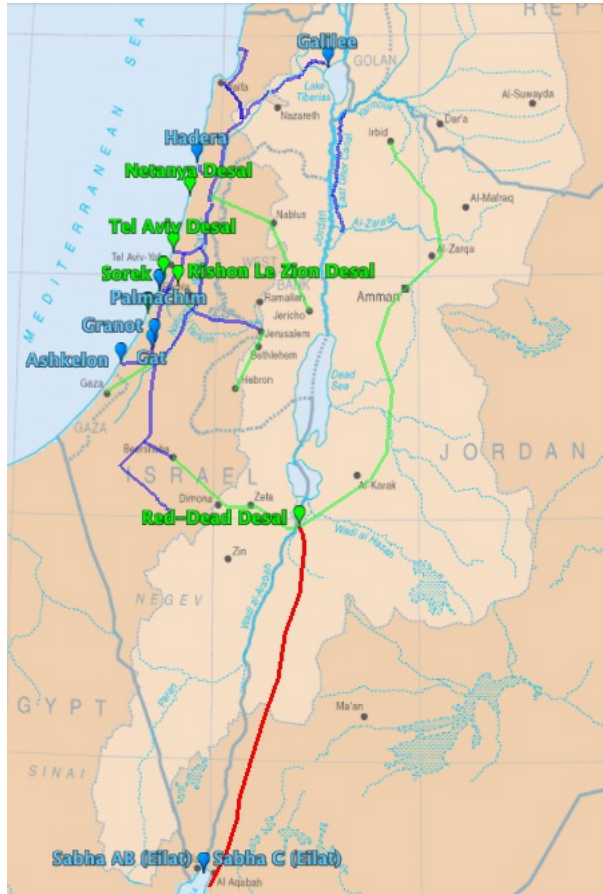


Figure 5: Minimum cost solution with equal per-capita demand. The existing network elements are blue, the network elements installed in this solution are green, and the Red-Dead Water Conveyance is shown in red.

more water to the Palestinian and Jordanian cities so that their per-capita water availability is equal to the Israeli cities (higher equity). This solution, seen in Figure 5, involves building the Red-Dead Water Conveyance to supply Jordan and southern Israel (Beersheba), and building desalination plants at Sorek, Palmachim, Ashdod, Tel Aviv, Reshon Le Zion, and Netanya to serve Israeli and Palestinian demand. The total cost of the “equitable” solution is \$1.8B

Thus by changing the demand parameters (the right hand side of the flow balance constraint) it is possible to set upper and lower bounds on total cost that correspond to maximum

and minimum levels of equity. The next step is to formulate a new model where the objective is to maximize “demand satisfied” subject a constraint on cost. This new model will be solved in increments in order to find intermediate balance between cost and equity.

Equity Model

Starting with the FCNF model, an additional variable q_i is added to be the proportion of the total equitable demand that is satisfied at node i . The objective function of the FCNF model now becomes a constraint that is set to be less than or equal to a value between the upper and lower cost bounds (*intvalue*). In addition, a minimum is set on q_i so that the proportion of total demand actually supplied at one node can not fall too low in relation to the other nodes. For example, this “floor” prevents the model from supplying only 25% of Amman’s water demand while supplying 100% of the demand of all the other cities, thus maintaining some level of equity. The algebraic formulation of the equity model is as follows:

$$\max \sum_{i \in N} q_i \quad (5)$$

subject to

$$\sum_{(i,j) \in A} (c_{ij}^U f_{ij} + h_{ij}^U u_{ij}) + \sum_{i \in N} (c_i^b y_i + h_i^b x_i) \leq \text{intvalue} \quad (6)$$

$$\sum_{j: (i,j) \in A} f_{ij} - \sum_{j: (j,i) \in A} f_{ji} = b_i q_i + b_i^+ y_i \quad i \in N \quad (7)$$

$$f_{ij} \leq U_{ij} + U_{ij}^+ u_{ij} \quad (i, j) \in A \quad (8)$$

$$y_i \leq x_i \quad i \in N \quad (9)$$

$$q_i \geq \text{minpercent} \quad i \in N \quad (10)$$

$$u_{ij} \in \{0, 1\}, (i, j) \in A; x_i \in \{0, 1\}, i \in V; 0 \leq y_i \leq 1, i \in V; 0 \leq q_i \leq 1, i \in V$$

The equity model is solved six times for intermediate values (parameter *intvalue*) between the upper bound on cost (the objective function value of the high cost - high equity solution) of \$1.8B and the lower bound (the objective function value for the low cost - low equity solution) of \$1.1B (rounded) by increments of \$100M. For all instances the *minpercent* of q_i is set to 0.65 as this is slightly better than the 60% of total equitable demand that the Jordanian cities receive in the min-cost min-equity solution.

Results

Below are the six solutions obtained for the intermediate cost limits with the new desalination plants, new water transmission connections, and cities that receive less than 100% of their “equitable” demand. In all solutions new connections were made from Amman to AzZarqa, AzZarqa to Irbid, from the NWC to Gaza, and from the western line of the NWC to the eastern line, so these connections are not listed in the tables below.

As the cap on the total cost is lowered, fewer desalination plants are built, and more of the cities are supplied with water from the Med-Dead desalination plant at the expense of the cities in Jordan. The figures associated with each of these solutions is included in the appendix.

These six alternative network solutions are obtained by varying the cost constraint while holding the minimum percentage constraint constant at 65%. More alternatives can be obtained by changing the minimum value of q_i (for example setting it at 90%).

In order to decide which is the “best” network solution, it would be necessary to elicit single dimension value functions for cost and for equity (q) from decision makers, have them set swing weights for these two attributes, and score each solution.

This chapter presented a method for optimizing the Middle East water distribution net-

Table 3: Solution at \$1.7B

| Desalination Plants | New Connections | Percent of Supply Received |
|---------------------|----------------------|----------------------------|
| Med-Dead | Med-Dead to Amman | Amman: 0.91 |
| Sorek | Jerusalem to Hebron | |
| Palmachim | Jerusalem to Jericho | |
| Ashdod | NWC to Nablus | |
| Haifa | Nablus to Jerusalem | |
| Rishon Le Zion | Hebron to Beersheba | |
| Ashdod II | | |
| Netanya | | |

Table 4: Solution at \$1.6B

| Desalination Plants | New Connections | Percent of Supply Received |
|---------------------|----------------------|----------------------------|
| Med-Dead | Med-Dead to Amman | Amman: 0.91 |
| Sorek | NWC to Hebron | |
| Palmachim | Hebron to Jerusalem | |
| Ashdod | Jericho to Jerusalem | |
| Tel Aviv | NWC to Nablus | |
| Haifa | Nablus to Jericho | |
| Rishon Le Zion | Nablus to Jerusalem | |
| Netanya | | |

Table 5: Solution at \$1.5B

| Desalination Plants | New Connections | Percent of Supply Received |
|---------------------|-----------------------|----------------------------|
| Med-Dead | Med-Dead to Amman | Amman: 0.78 |
| Sorek | Med-Dead to Beersheba | |
| Palmachim | Med-Dead to Jericho | |
| Ashdod | Jerusalem to Hebron | |
| Tel Aviv | NWC to Nablus | |
| Rishon Le Zion | Nablus to Jerusalem | |
| Netanya | | |

work for two competing objectives. It is not possible to optimize cost and equity at the same time, so it is necessary to find alternatives that reach a compromise between the two objectives. The optimization procedure presented here could be used in conjunction with

Table 6: Solution at \$1.4B

| Desalination Plants | New Connections | Percent of Supply Received |
|---------------------|-----------------------|---------------------------------|
| Med-Dead | Med-Dead to Amman | Israel agricultural water: 0.84 |
| Sorek | Med-Dead to Beersheba | Amman: 0.65 |
| Palmachim | Med-Dead to Jericho | |
| Shomrat | Med-Dead to Hebron | |
| Tel Aviv | Jerusalem to Hebron | |
| Rishon Le Zion | NWC to Nablus | |
| Ashdod II | Nablus to Jerusalem | |

Table 7: Solution at \$1.3B

| Desalination Plants | New Connections | Percent of Supply Received |
|---------------------|-----------------------|---------------------------------|
| Med-Dead | Med-Dead to Jerusalem | Israel agricultural water: 0.65 |
| Sorek | Med-Dead to Amman | Amman: 0.65 |
| Palmachim | Med-Dead to Beersheba | AzZarqa: 0.78 |
| Ashdod | Med-Dead to Jericho | |
| Shomrat | Med-Dead to Hebron | |
| Rishon Le Zion | NWC to Nablus | |
| Ashdod II | | |

Table 8: Solution at \$1.2B

| Desalination Plants | New Connections | Percent of Supply Received |
|---------------------|-----------------------|----------------------------------|
| Med-Dead | Med-Dead to Jerusalem | Israeli agricultural water: 0.65 |
| Sorek | Med-Dead to Amman | Amman: 0.65 |
| Palmachim | Med-Dead to Beersheba | AzZarqua: 0.65 |
| Ashdod | Med-Dead to Jericho | Irbid: 0.65 |
| Rishon Le Zion | Med-Dead to Hebron | Gaza: 0.87 |
| | NWC to Nablus | |

a full multi-attribute decision analysis process (identify objectives, develop evaluation measures, create single dimension value function, weigh objectives, etc.) by Middle East decision makers in order to find a solution that best meets the various objectives of all the people in the region.

Chapter 3: Incorporating Uncertainty

Incorporating Uncertainty into Linear Programs

An MIP assumes deterministic input parameters (that is, known costs and demands), but in this problem a water distribution system is being optimized to meet current and uncertain future demands. There is uncertainty as to the actual future demands, there is uncertainty as to the cost of building infrastructure (the price of construction materials change all the time, the interest rates change), and there is uncertainty as to the operating costs (labor costs can change, energy prices can fluctuate). For these reasons, an MIP using deterministic inputs of costs and demands may not accurately model the real-world situation, and therefore not provide a realistic answer.

Other researchers have used math programming and optimization techniques to solve water distribution problems. A common approach is to formulate the design of a water distribution network as a single-stage optimization model. These models have evolved to incorporate deterministic measures of pipe reliability and maintenance cost. A number of investigators have employed heuristic procedures for single-stage optimization models for water distribution. However, a common weakness of these approaches is that the sole use of deterministic optimization fails to properly incorporate uncertainty into the models. The following review covers possible methods to incorporate uncertainty into math programs in

general and network programs specifically.

Stochastic Programming

Sen and Higle (1999) present a tutorial of linear-programming models for optimization under uncertainty, which they call stochastic linear programming (SLP). They point out the pitfalls of naïve approaches to SLP, such as using expected values of random variables as inputs or using the “wait and see approach,” where the LP is solved for each possible outcome of the random variable. In each of these cases, the solution may be infeasible in respect to alternative outcomes of the random variables, limiting the usefulness of this type of analysis. As an alternative, Sen and Higle present two-stage recourse models where some decision variables are implemented before the outcome of the random variable is observed and some decision variables are implemented after.

Simple recourse models compensate for violations of the right-hand side of the constraints by adding a proportional penalty in the objective function. The simple recourse method allows for first-stage (planning) decisions that may not satisfy all the constraints to still be acceptable, just more costly. Simple recourse is still limited though, as it is not flexible in its response to uncertainty — recourse actions are restricted to imposing a penalty in the objective function. The general recourse model, on the other hand, divides the problem into two stages, random variables that must be decided right away, and random variables whose decision can be postponed. Instead of adding a penalty to the objective function, the constraints are re-written to include constraints that included only the first stage decision variables, and constraints that include both sets of decision variables.

In both the simple and general recourse models, a cost must be assigned to the “recourse action” (the penalty for violating feasibility), but in some models it may be more realistic to allow a constraint to be violated, as long as the probability of violating the constraint is

held below some threshold. For example, in models of network reliability or service times, it may be a given that the system can not be designed to meet its requirements under all circumstances – 100% of the time – rather the system is designed to meet demand “most” of the time.

These types of models can be formulated again as an extension of deterministic linear programs, but with probabilistic constraints. An example of how this could be formulated is given by Sen and Higele (1999) as follows, with $A_1x \geq b_1$ being the constraint that only contains deterministic variables, and $P(\tilde{A}_2x \geq \tilde{b}_2) \geq p$ being the probabilistic constraint. The linear program is then formulated as follows:

Minimize cx
subject to

$$\begin{aligned} A_1x &\geq b_1 \\ P(\tilde{A}_2x \geq \tilde{b}_2) &\geq p \\ L &\leq x \leq U \end{aligned}$$

In this example $p \in (0, 1)$ is the threshold reliability required of the system.

Robust Optimization

A different approach to dealing with uncertainty in mathematical programming is to focus on the tradeoffs between the means and variances of the objective. This type of model may be appropriate when one wishes to minimize the variance of the objective value or to optimize the worst-case given the data. Sen and Higele (1999) and Bertsimas and Sim (2003) point out that many of the earlier approaches to robust optimization find solutions that are over-conservative, and Sen and Higel recommend against the method. However, Bertsimas and

Sim present a robust optimization approach that allows them to control how conservative the solution is by setting probabilistic bounds on the constraint violations specifically in discrete optimization and network flow problems. Their method allows for the model to be adjusted to control for the tradeoff between optimizing the objective or increasing the robustness by varying a single model parameter.

Simulation Optimization

Two stage or multi-stage stochastic programming may be appropriate for integrating random variables with discrete distributions or for modeling a limited number of outcomes, but when the random variables have a continuous probability distribution or there are many variables in the model, it may be prohibitive to enumerate all the possible scenarios. A solution to this problem is to replace the random variables in the model with a sample taken from the probability distributions of the random variable. The model can then be solved n times, with n being the number of samples drawn from the distributions. It has been shown that the optimal solutions obtained will converge as the number of samples increases.

Incorporating Uncertainty in Network Models

Dye et al. (2003) Dye et al. (2003) present a stochastic programming example of a network optimization problem where the objective is to maximize profit in a telecommunications when capacity is limited and demand is uncertain. They demonstrate that while a polynomial approximation scheme is known for the deterministic version of the problem, making the demands stochastic makes the problem strongly NP-hard. The addition of stochasticity results in a worse-case performance ratio that increases as the number of input parameters increases. The authors find that if they limit the number of instances they can develop a

heuristic with a constant worst-case performance ratio.

Gutierrez et al. (1996) use a robust optimization approach to find optimal designs in uncapacitated networks. The authors use an adaptation of Benders decomposition methodology and tailor it for efficiency. This algorithm has the advantage that it considers information about all the data scenarios simultaneously and results in an adequate number of robust network designs. However, this method is only applicable to situations with a pre-specified set of future operating scenarios.

Gürkan et al. (1999) use the sample-path technique to solve optimization problems when the objective function and constraints are both stochastic. They apply their new technique to a network design problem to find optimal arc capacities by minimizing the sum of capacity allocation costs and a measure of the expected shortfall in capacity when supply and demand are uncertain.

Recent work in water distribution optimization mainly involves simulation optimization techniques that incorporate various meta-heuristics such as genetic algorithms, simulated annealing, ant colony optimization, shuffling frog leaping algorithms, and particle swarm optimization. For example, Eusuff and Lansey (2003) use the shuffling frog leaping algorithm combined with hydraulic simulation software EPANET (U.S. EPA, 2009) to select optimal pipe diameter sizes in a two-looped process, whereas Suribabu and Neelakantan (2006) take a similar approach but find particle swarm optimization to be more efficient than the shuffling frog leaping algorithm.

The approach used in this research is similar to previous work on finding shortest paths in stochastic graphs; i.e., graphs where the arc costs are represented by random variables with possibly known probability distributions. Frank (1969) presents a method to compute the probability distributions of shortest path lengths in such graphs using Monte Carlo simulation and to compare paths according to the probabilistic costs. Sigal et al. (1980)

develop a new performance measure called a path optimality index (the probability that a path is shorter than all other paths) and present a procedure using the integration of the uniformly directed cutsets to compare of all candidate paths.

This chapter explains the process for specifying distributions for the uncertain input parameters (costs, demands, etc.), sampling the distributions in a Monte Carlo fashion, and employing a Bayesian model selection framework to find the most likely minimum cost network design.

Bayesian Mixed Integer Programming

The procedure uses concepts from Bayesian Model Averaging (BMA) to characterize the distribution of solutions to an MIP with random inputs. This approach is called Bayesian Mixed-Integer Programming (BMIP). Background information on BMA and Bayesian analysis via sampling methods can be found in (Hoeting et al., 1999), gelman:03,gilks:06.

Suppose an MIP is given with unspecified values for inputs and whose objective function reflects a desire to minimize cost. Inputs include objective function coefficients, constraint coefficients, right-hand sides of inequalities, and variable bounds. Also given are the data that reflects the uncertainty in the input values. The output of the BMIP procedure includes:

1. the probability that a set of values for the discrete decision variables in the MIP are optimal for a realization of the inputs,
2. the probability that a certain value for a discrete decision variable in the MIP occurs in a least-cost solution,
3. the distribution of optimal values for continuous decision variables in the MIP, and
4. the probability that a given set of values for the discrete decision variables is feasible for a realization of the inputs for the MIP

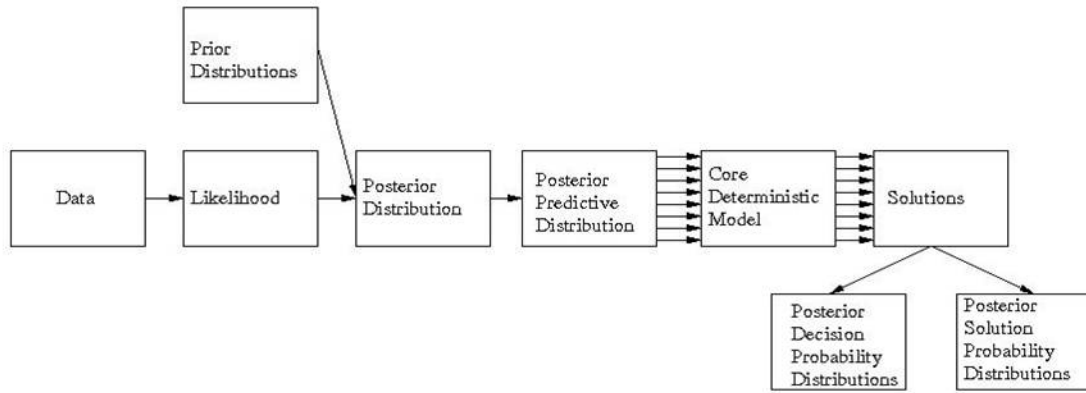


Figure 6: Steps in the Bayesian Mixed Integer Programming process

Figure 6 illustrates the steps involved in the process. The data are combined with pre-specified likelihood functions for the data and with the prior distributions for the parameters of the likelihood functions to form the joint posterior distribution. The posterior distribution, in conjunction with the likelihood of a new input value, is used to create the joint posterior predictive distribution of new inputs to the MIP. The posterior predictive distribution is sampled repeatedly and passed to the MIP and an optimal solution for each sample is obtained. Posterior *solution* probability distributions can then be empirically derived from the set of all solutions, characterizing the distribution of candidate sets of values for discrete decision variables in a least-cost solution. Additionally, posterior *decision* probability distributions can be derived from the set of all solutions, characterizing the distribution of variable values for individual decision variables. The posterior solution probabilities can be used to compare candidate networks and select most-probable least-cost values for the discrete decision variables.

Instead of simply using the sample mean, \bar{a} , as a point estimate of an individual input to the MIP, the BMIP process uses a sample from the posterior predictive distribution for a

future value of the input, a^* , given the data D , which comes from:

$$p(a^*|D) = \int_{\Theta} p(a^*|\theta)p(\theta|D)d\theta$$

where $p(a^*|\theta)$ is the likelihood for a^* given θ and $p(\theta|D)$ is the posterior distribution as found by Bayes theorem:

$$P(\theta|D) = \frac{P(\theta)P(D|\theta)}{\int_{\Theta} P(S_i|\theta, D)P(\theta|D)d\theta}$$

where $P(\theta)$ is the prior distribution of θ and $P(D|\theta)$ is the likelihood of the data given θ .

Evaluating $p(\theta|D)$ and $p(a^*|D)$ can be achieved via a Markov-Chain-Monte-Carlo sampling algorithm such as Gibbs sampling or Metropolis-Hastings sampling. These will generate samples from the posterior predictive probability distributions for the MIP inputs so that the range of behaviors of input is captured and their effect on optimal MIP solutions can be observed.

Modeling Input Parameters

The core model is the same fixed cost network flow, mixed-integer program (MIP) model described in Chapter 2 (equations 1 - 4). However, instead of using single point estimates for the model parameters, to use the BMIP process it is necessary to generate probability distributions for each input parameter. These distributions are the Bayesian posterior predictive distributions as described below.

Modeling Demand

Dynamic Linear Model for population growth

Modeling the demand for water in the year 2025 involves several steps. First, it is necessary to determine the the population growth rate for each country individually, as for political, social, and demographic reasons, the populations of Israel, Jordan, and the Palestinian Territories are expected to be growing at different rates. The growth rate can be determined by the following:

$$r_t = \frac{N_t - N_{t-1}}{N_{t-1}}$$

where N_t is the size of the population at time t . Population data from the last 18 years (1990 to 2008) are obtained from the World Bank database for Israel, Jordan, and the Palestinian territories.(The World Bank, 2010) Instead of just using the mean growth rate from this period, the data are fit to a simple Dynamic Linear Model (DLM) given by:

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t, \\ \mu_t &= \mu_{t-1} + \eta_t \end{aligned}$$

where r_t is the growth rate and μ_t is the mean growth rate at time t , $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $\eta_t \sim N(0, \sigma_\eta^2)$. Thus, instead of just calculating a single growth rate at time t , the dynamic linear model calculates the growth rate as a normal distribution with a mean growth rate plus variance. The “fit” of the model to the data can be adjusted by modifying the prior probabilities (the σ_ε^2 and the σ_η^2). Ideally the 95% credible interval of the posterior distribution is wide enough to capture the data points, but not too wide as this will increase the variation in the posterior predictive distributions. After the model is fit, posterior predictive distributions can be calculated for time $t + 1$, $t + 2$, etc. The variance of the posterior predictive distributions grows with each time period in the future as the variance is cumulative and

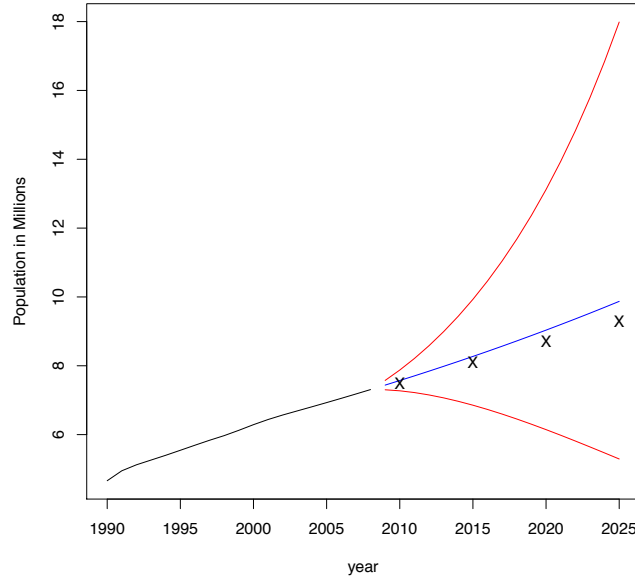


Figure 7: Population forecast from DLM model for Israel (in blue) with upper and lower bounds of the 95% credible predictive interval (in red), and the Israeli government's population projection for 2010, 2015, 2020, and 2025 (Xs)

dependent on the variance of all the previous predictions. The advantage to using a dynamic linear model to create a distribution rather than a point estimate is that the distribution can be sampled to capture the range of possible future outcomes. In this case, the posterior predictive distributions of the growth rates were sampled 100,000 times for each year up to 2025. Then using the starting population data (year $t = 2008$), estimates of the population for each successive year are forecasted by taking the population in year t and multiplying it by $(1 + r_t)$ to get the population in year $t + 1$.

As seen in Figure 7, the population forecast for Israel, the 95% credible predictive interval grows wider each year in the future as the uncertainty increases. Although the range of the population predictions in 2025 is large (from no change in population to 3 times current population, because the posterior predictive distributions are normally distributed, the

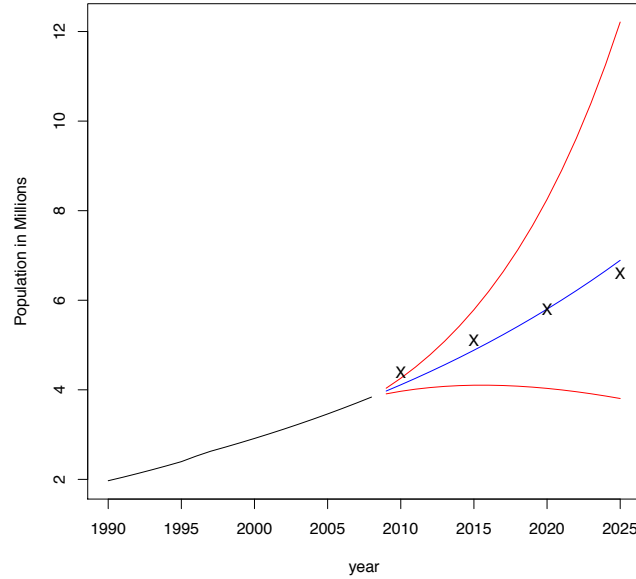


Figure 8: Population forecast from DLM model for the Palestinian Territories (in blue) with upper and lower bounds of the 95% credible predictive interval (in red), and the United Nation's population projection the West Bank and Gaza in 2010, 2015, 2020, and 2025 (Xs)

probabilities of the extreme predictions are low. The forecast from the DLM is fairly well calibrated to the population forecasts for the years 2010, 2015, 2020, and 2025 published by the Israel Central Bureau of Statistics (Israel Central Bureau of Statistics, 2010). The population forecast for the Palestinian Territories can be seen in Figure 8 and the forecast for Jordan can be seen in Figure 9. While the DLM forecast for the Palestinian Territories is well aligned with population projections published by the United Nations (United Nations Statistics Division, 2008), the DLM forecast for population projection of Jordan is somewhat higher than the United Nations' (United Nations Statistics Division, 2008).

The next step in calculating future water demand is to estimate the per-capita water demand rate in cubic meters (m^3) per person per year. In each country, the majority of water is consumed in the agricultural sector (58% in Israel (Fixler, 2010)), but the assumption is

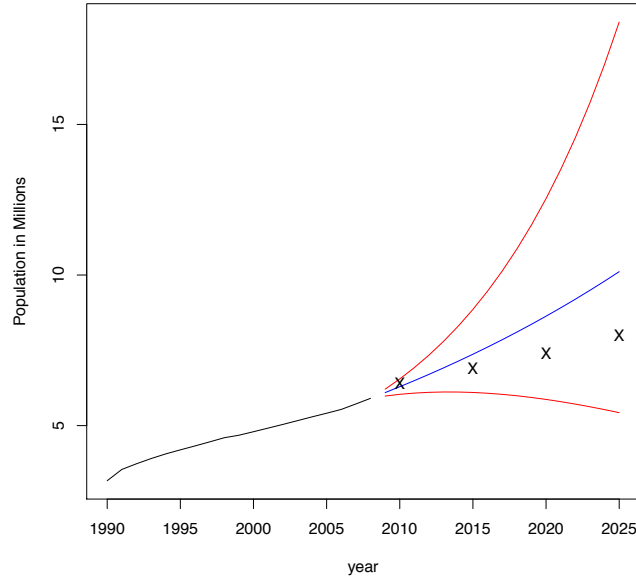


Figure 9: Population forecast from DLM model for Jordan (in blue) with upper and lower bounds of the 95% credible predictive interval (in red), and the United Nation’s population projection for Jordan in 2010, 2015, 2020, and 2025 (Xs)

that any water supplied by desalination plants or the large conveyance projects will be primarily for domestic uses. It is assumed that agricultural use of “fresh” water will be capped a current usage rates (as is the current policy in Israel), and any growth in agricultural demand will be met by increasing the use of brackish water and treated waste water, and only the demand for “fresh” water is considered in this optimization problem.

Data for current domestic water consumption comes from several sources. According to Fixler (2010), the mean annual per-capita domestic water consumption in Israel for the last 11 years (1997-2008) was 104.67 cubic meters per person per year with a standard deviation of 9.33 cubic meters. From this data a normal distribution was constructed and 100,000 samples of consumption rates were taken and multiplied times the 100,000 samples of population size to get 100,000 predictions of domestic water demand for Israel. Similarly,

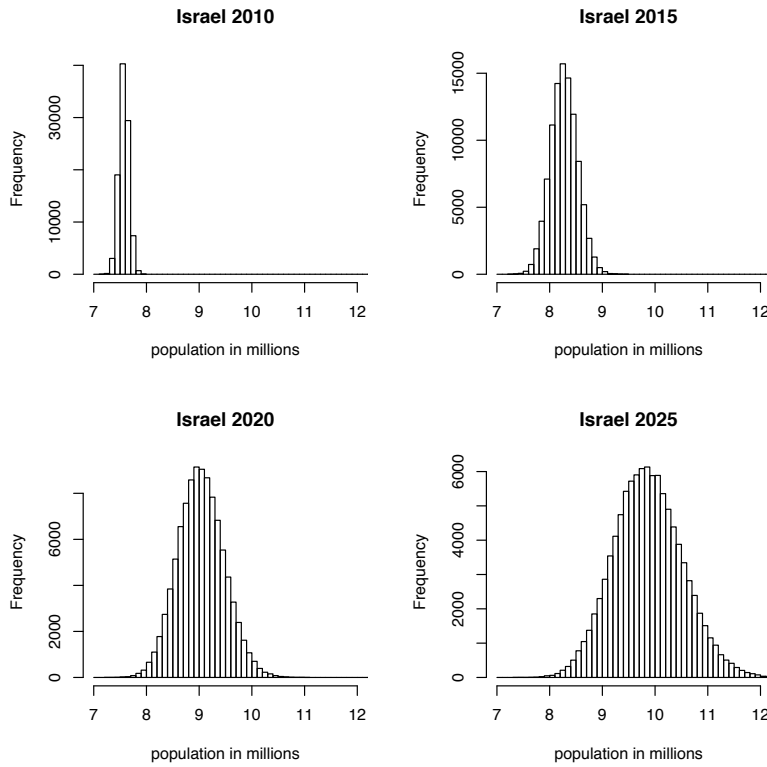


Figure 10: Histogram of Israeli population predictions from 2010 to 2025

the domestic water demand of Jordan was predicted by the same procedure but assuming a normal distribution of per capita water consumption with a mean of 60.23 cubic meters and a standard deviation of 7.61 cubic meters. Although it is accepted that the per capital domestic consumption in the Palestinian Territories is much less than in Israel or Jordan, data for the Palestinians were limited. A triangular distribution was assumed using 3 rates found from different sources (Rabi, 2009; Palestinian National Authority and Palestinian Water Authority), with a low of 26.73, a high of 49.35, and a mode of 34 cubic meters per person per year. Again, this distribution was sampled and multiplied times the Palestinian population forecasts to predict the Palestinian water demand.

After the total population and resulting water demand was forecasted, the demand had

to be allocated to the demand points in the model. Websites for each nation (Israel Central Bureau of Statistics, 2010; The Hashemite Kingdom of Jordan Department of Statistics, 2010; Palestinian Central Bureau of Statistics, 2010) were consulted to find the current distribution of population by district. Since the network representing the problem area does not have a node in each district, the districts were grouped around the closest node and the entire proportion of the grouped population assigned to that district. For example, Israel divides its nation into 7 districts and 16 subdistricts, yet the network representation of Israel only has 13 demand points, so some of the subdistricts were combined and are represented as one demand point. The distribution of current population for the districts of Israel can be seen in Table 15 in the Appendix.

The distribution of population is not likely to be exactly the same in 2020 as it is today, some cities will gain proportion, some will decline, and the demand for water is not likely to be exactly proportional to the population of each district. For these reasons, using the Dirichlet distribution to allocate water demand to each node is helpful. The Dirichlet distribution can be used to generate variability in any type of “string cutting” problem, that is, any problem where a whole has to be cut into several pieces, of slightly varying proportions, but that always sum to the whole. The Dirichlet distribution is formulated as follows:

$$p(\varphi_1, \varphi_2, \dots, \varphi_{k-1} | N_{t,1}, N_{t,2}, \dots, N_{t,k}) \propto \prod_{i=1}^k \varphi_i^{N_{t,i}-1}$$

where $N_{t,i}$ is the current proportion of the total population at node i . A uniform Dirichlet distribution is specified for φ_i and 100,000 samples are taken of population proportion. Since the Dirichlet distribution has the property that $\sum_{i=1}^k \varphi_i = 1$, the demand will be distributed across the demand points while maintaining the total demand.

Agricultural demand for freshwater was handled differently for Israel and Jordan and the

Palestinian Territories. According to Fixler (2010) the agricultural allocation of fresh, natural water from the NWC will be capped at 450 MCM per year from now on. In the optimization model this demand is held constant and directed to one demand point in southern Israel. The situation in Jordan and the Palestinian Territories is slightly different, where most of the water is supplied from ground water wells. The combined domestic and agricultural demands are already creating a shortage of supply resulting in over-extraction (removal of water from aquifers or streams above sustainable levels). It is not possible to determine how much of the shortage is attributable to domestic demand or agricultural demand, but is assumed that any new water supply and distribution system will have to satisfy the current shortage (so that domestic use of ground water can be shifted to meet the agricultural demand) in addition to meeting the projected increase in domestic water demand from year t to year $t+1$. Therefore the current shortage is used as a starting point (to capture the agricultural demand) and only the increase in water demand each year is added to the total water demand for Jordan and the Palestinian Territories.

The resulting inputs into the optimization model are samples from the posterior predictive distribution of the total annual demands at each demand node i at time t , generated by

$$b_{it}^* = \varphi_i^* N_i^* \varpi^*$$

where φ_i^* is a sample from the i^{th} component of the allocation vector distribution, N_i^* is a sample from the posterior predictive distribution of the population, and ϖ^* is a sample from the posterior predictive distribution of per-capita water consumption.

Modeling Costs

Modeling Desalination Plant Costs

The objective of the optimization model is to minimize the total cost of producing and transporting water to the demand locations. Two natural categories of expenses are the capital costs of building the desalination plants and infrastructure, and the operational costs of running the plants and transporting the water. To make a fair comparison among the options, it is necessary to compare the lifetime costs of the desalination plants, and not just their investment costs. Take the following example of three desalination plant options, assuming they all have the same output in MCM per year:

Table 9: Desalination plant example

| Plant | A | B | C |
|----------------------|------------|-----|-----|
| Investment Cost | 300 | 500 | 400 |
| Annual O and M Costs | 40 | 20 | 10 |
| Sum | 340 | 520 | 410 |

If we just summed the investment and operations and maintenance costs, it would appear that plant A, with a total cost of \$340 million dollars is the best investment; however, this is not a fair comparison, because it does not take into account the cumulative O&M costs over the lifetime of the plant nor the fact that the capital investment is financed over many years. If we assume that the lifetime of a desalination plant (and the repayment period of financing the construction of the plant) is 25 years, we can calculate the annualized capital expenses by multiplying the investment cost with the annuity factor

$$a = \frac{i}{1 - \left(\frac{1}{1+i}\right)^n}$$

where i is the interest rate (discount rate) and n is the lifetime of the desalination plant.

Using an interest rate of 8%, the following annual capital costs can be calculated and then the total annualized cost can be compared:

Table 10: Example with annualized capital costs

| Plant | A | B | C |
|---------------------|------|------|-------------|
| Investment Cost | 28 | 47 | 37 |
| Annual O&M Costs | 40 | 20 | 10 |
| Total annual cost | 68 | 67 | 47 |
| Lifetime cost (x25) | 1703 | 1671 | 1187 |

It is clear now that plant A is actually the worst option when the lifetime cost of the plants are compared. While the choice of interest rate does influence the relative annual costs, in this example, plant C will have the lowest annualized (and lifetime) cost until the interest rate reaches 30%.

The costs of desalinating water is usually reported in the unit cost of water, which is the annualized capital and O&M expenses divided by the annual output of the plant (m^3 per year). According to the International Desalination Association, (Global Water Intelligence, 2009) the typical cost breakdown of the annual cost of water is as seen in Table 11.

Table 11: Breakdown of annual desalination plant costs

| | |
|--------------------------|-----|
| Annualized Capital Costs | 37% |
| Energy (at \$0.07/kWh) | 36% |
| Chemicals | 12% |
| Total Maintenance | 6% |
| Total Labor | 4% |
| Membranes (5 yr. life) | 4% |
| Cartridge Filters | 1% |

Since most of the data on desalination costs are given as unit costs, predictions of unit costs of the potential desalination plants can be used to extrapolate the total capital and operation and maintenance costs based on the plant's output capacity. Data from 44 large

scale desalination plants installed in the last decade were used to create a non-linear regression model of unit cost by capacity (Global Water Intelligence, 2009). The nonlinear regression was approximated using the R Statistical Package (Ihaka and Gentleman, 1996), and then samples of the posterior distribution of the regression parameters were generated in WinBugs (Lunn et al., 2000). Figure 11 shows the fitted regression model on the data.

$$Y_i = \alpha - \beta(1 - e^{-x_i/\lambda}) + \epsilon_i, i = 1, \dots, n,$$

where Y_i is the unit cost of plant i , x_i is the capacity of plant i , and $\epsilon_i \sim N(0, \tau)$. Normal prior distributions are set for α and β ($\alpha \sim N(1.6, 0.0001)$ and $\beta \sim N(0.8, 0.0001)$); a gamma distribution is set for λ ($\lambda \sim \gamma(18, 0.8)$) and a gamma distribution is given to τ ($\gamma(0.1, 0.1)$). If the prior distributions are sampled a sufficient number of times, the resulting joint distribution is a good approximation to the true posterior distribution of the model parameters. Using WinBugs, 100,000 samples were created for the model parameters (α, β , and λ) using the prior distributions and initial values. This constructed posterior distribution approximation can then be used to generate 100,000 predictions of the unit costs of a potential desalination plant Y_i^* by solving the regression model with the plant's annual capacity, x_i^* .

Once the unit cost predictions are obtained, they are split into annualized capital costs and annual operating costs based on the International Desalination Association's estimate that annualized capital expenses make up 37% of the unit cost and annual operating expenses make up 63% of the unit cost. The Dirichlet distribution is used again to generate variation on proportion of capital expenses to operating expenses.

$$p(v_1, v_2 | U_1, U_2) \propto \prod_{i=1}^2 v_i^{U_i-1}$$

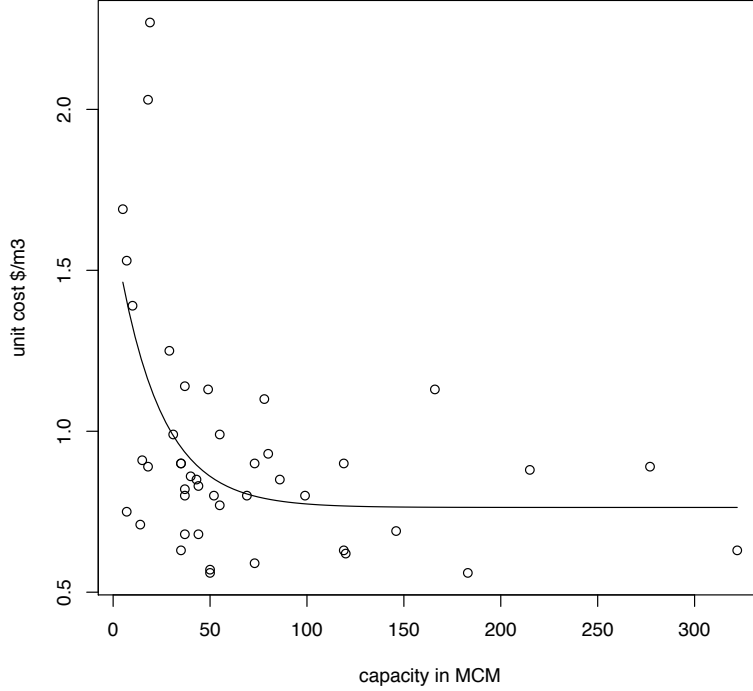


Figure 11: Regression model plotted on a scatter plot of the unit cost data.

where $U_1 = 0.37$ and $U_2 = 0.67$.

Samples of the annual capital costs for each desalination plant ξ_i^* are then obtained by multiplying $v_{1,i} * Y_i^*$. However, this does not include the expense of the Red-Dead Water Conveyance or Med-Dead Water Conveyance. To estimate the annualized costs of building these large projects Ξ_i , an estimate of the initial cost of each project is multiplied times an annuity factor. The annuity factor is estimated using a time period of 50 and an interest rate $\sim U(0.03, 0.08)$, and an initial investment is given a triangular distribution with at low of \$4 billion, a high of \$7.5 billion, and a median of \$6 billion (based on estimates found in The World Bank (2007) Israel Ministry of Foreign Affairs (2002)). Then the annual capital

costs for the two water conveyance and desalination projects are:

$$\xi_{MedDead} = \xi_{MedDead} + \Xi_{MedDead}$$

$$\xi_{RedDead} = \xi_{RedDead} + \Xi_{RedDead}$$

The annual operating costs for new plants can similarly be obtained by multiplying $v_{2,i} * Y_i^*$, whereas the operating costs for the existing plants are taken from the literature (Global Water Intelligence, 2009).

Modeling the Cost of Water Transmission Lines

Data on the costs of installing water transmission lines is not abundant in the literature. Zhou and Tol (2004) note that most engineering firms consider this information proprietary, and are not willing to share it with researchers. However, Kally (1993) does present estimates the cost of moving water long distances in with a specific focus on projects in Egypt, Israel, and Turkey. Kally estimates the costs of moving water to be \$610 for every kilometer of horizontal distance, and \$520 for every meter in lift per MCM (million cubic meters). This cost can be considered similar to the unit cost of water; it is composed of a mix of annualized capital costs and annual operation and maintenance costs. For simplicity then, the installation cost of adding new capacity is considered to be \$610 per m^3 per kilometer, and the operation and maintenance costs to be a flat \$10 per m^3 per kilometer and \$233 per m^3 per m elevation gain. The “pumping cost” is as an estimate of the energy costs of pumping water (at \$0.06 per kWh), as calculated using the tutorial in Peacock (1998). Unfortunately there is not enough data to perform a Bayesian data analysis to incorporate uncertainty in the water transmission costs.

Modeling Environmental Costs

At the operational phase, the most significant aspects of supplying water from a desalination plant are energy use (impact = air emissions from electricity generation - CO₂, SO_x, NO_x) and the brine generation and discharge (impact = decline in water quality). The most significant aspects of distributing the water are energy use for pumping (impact = air emissions from electricity generation - CO₂, SO_x, NO_x) and (in the case of the Med Dead and Red Dead projects) the potential for salt water to leak from the transmission pipes/ canals.

Of the aspects and impacts identified, energy consumption and the resulting air pollution from desalination plants was decided to be the most straightforward to model. Empirical prior distributions for the cost of CO₂ emissions, ν , that results from an associated amount of energy, ε , needed to desalinate a m^3 of water are as defined as $\mu_\nu \sim N(\hat{\mu}_\nu, 10)$, $\mu_\varepsilon \sim N(\hat{\mu}_\varepsilon, 10)$, $\sigma_\nu^2 \sim Inv - \chi^2(6, 0.1)$ and $\sigma_\varepsilon^2 \sim Inv - \chi^2(6, 0.1)$, where $\hat{\mu}_\nu$ and $\hat{\mu}_\varepsilon$ are respective sample means. Data on carbon emissions, ν_i , was based on Israel's average emission factor of CO₂ from electricity generation Israeli Electric Corporation (IEC) (2009). Energy requirements, ε_i , was set to be a recent average of the cost a European Allowance Unit (EAU) on the European Carbon Exchange (ECX), where an EAU is "issued to installations which have a cap on their emissions under the EU Emission Trading Scheme. Each EUA grants the installation the right to emit one tonne [metric ton] of carbon dioxide during a commitment period." (European Climate Exchange, 2009) Using the sample data the following likelihoods were generated: $\nu_i \sim N(\mu_\nu, \sigma_\nu^2)$ and $\varepsilon_i \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$. Using Bayes theorem, samples ν^* and ε^* can be obtained from their respective posterior predictive distributions.

Quantifying the cost of an adverse environmental impact can be difficult when the natural resource or ecosystem does not have a direct market value. In these circumstances contingent valuation, travel costs, hedonic pricing, and habitat equivalency analysis can be used to quantify environmental impact; although not part of this study, one avenue for future

research could be to use one of these methods to quantify the environmental impact of the brine discharge and the risk of salt-water pipes breaking.

Results of Deterministic Model

The solution to the fixed cost network flow mixed-integer program (MIP) model described in Chapter 2 (equations 1 - 4), using point estimates for costs and demands, is to use all of the existing supply nodes in Israel (except the Hadera plant) to capacity and add the Med-Dead Water Conveyance, the Soreq plant, and the addition to the Palmachim plant. To meet water demand in Jordan, a new connection is added from the Med-Dead desalination plant to Amman, and from Amman to AzZarqa, and then from AzZarqa to Irbid. To supply water to the Palestinians, connections are built from the Med-Dead plant to Jerusalem & East Jerusalem, Jericho, and Hebron; Nablus and Gaza are supplied by new connections to the Israeli National Water Carrier (NWC). Water is also sent from the Dead Sea desalination plant to Beersheba. A graphical representation of this solution can be seen in Figure 12.

An alternative solution is obtained if we change the assumptions, for example, if Israel does not want to participate in a cooperative regional plan and only considers their own future water demands. In this case, the optimal solution is not to build the Med-Dead Water Conveyance and desalination, and instead build new desalination plants at Sorek and the addition to Palmachim.

When the Jordanians consider only their future water needs, the optimal solution is to use the Med-Dead Water Conveyance and desalination plant at 59% of capacity. However, given that the Med-Dead Water Conveyance is completely in Israeli territory, it is unlikely this solution is politically feasible. The more likely solution is for the Jordanians to use the Red-Dead Water Conveyance and desalination plant at 55% of capacity.

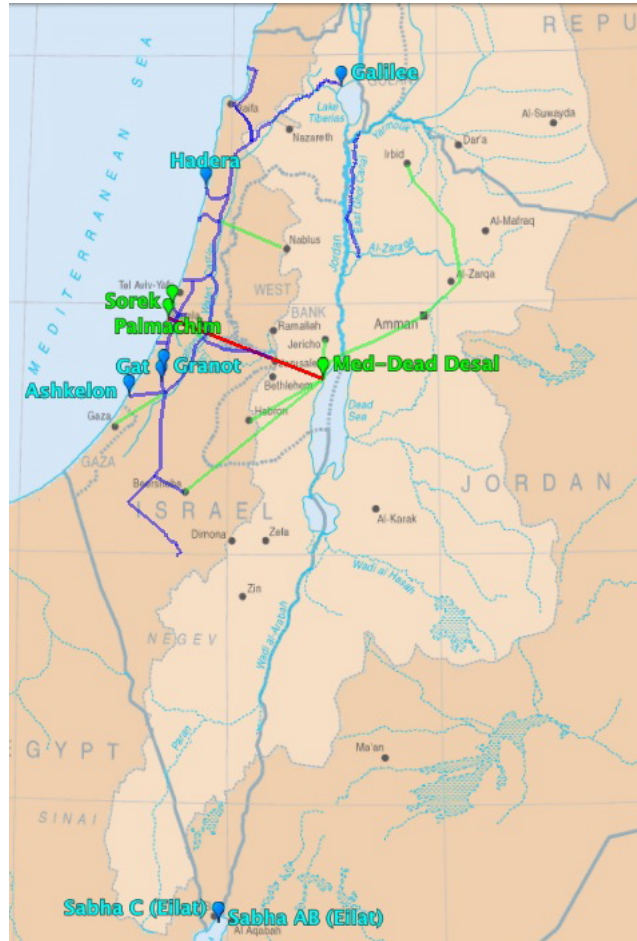


Figure 12: Solution to the deterministic model. The existing network elements are blue, the network elements installed in this solution are green, and the Med-Dead Water Conveyance is shown in red.

Results of Bayesian Mixed Integer Program

Most Probable Solution

The BMIP methodology is applied to the Middle East water distribution problem to generate solutions that reflect uncertainty in demand and costs. A solution to the BMIP consists of the values of the x_i variables (decision whether or not to install desalination plant i) and the values of the f_{ij} variables. The values of the f_{ij} variables are converted into binary values (if

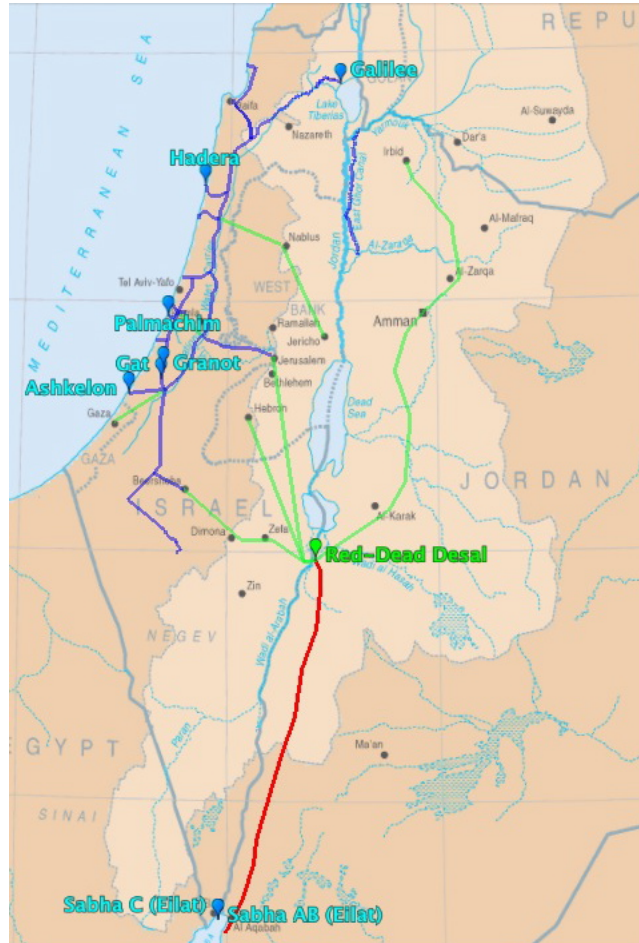


Figure 13: Most probable solution from the BMIP. The existing network elements are blue, the network elements installed in this solution are green, and the Red-Dead Water Conveyance is shown in red.

flow on arc $i, j > 0$, then $f_{ij} = 1$) so that the unique solutions can be aggregated and counted. When the 100,000 solutions consisting of the binary values of the desalination plants and new arcs are aggregated, there are 1824 unique combinations of desalination plants and arcs that are “optimal solutions.” 15% of the simulations result in no feasible solution, and the most probable (most frequently occurring) least cost solution has a frequency of 4%.

The most probable solution is to build only the Red-Dead Water Conveyance and desalination plant (no additional new desalination plants), and build new connections from the

Table 12: Probability that a desalination plant is included in the least cost solution

| | | | | | |
|-----------|-----|----------------|-----|-----------|-----|
| Soreq | 32% | Med-Dead | 56% | Ashdod II | 10% |
| Palmachim | 15% | Tel Aviv | 6% | Red-Dead | 44% |
| Ashdod | 22% | Haifa | 2% | Netanya | 3% |
| Shomrat | 11% | Rishon Le Zion | 32% | | |

Red-Dead desalination plant to Jerusalem, the Red-Dead desalination plant to Beersheba, the Red-Dead desalination plant to Hebron, the Red-Dead desalination plant to Amman, Amma to AzZarqa, AzZarqa to Irbid, the NWC to Gaza, the NWC to Nablus, and from Nablus to Jericho. The graphical representation of the most-probable solution can be seen in Figure 13.

Component Probabilities

In addition to providing the probability of the solutions, the BMIP output can be used to find the individual component probabilities, and the probabilities of unique sets of components (desalination plants or new connections). For example, the probability that each desalination plant is included in an optimal solution are seen in Table 12. While the most probable solution includes the Red-Dead Water Conveyance and desalination plant, the Med-Dead Water Conveyance and desalination plant actually occurs in more of the optimal solutions: 56% of the solutions include the Med-Dead Water Conveyance and desalination plant and 44% of the solutions include the Red-Dead Water Conveyance and desalination plant. Additionally, the solutions can be segmented to consider just the set of desalination plants without the new connections. The number of unique solutions then decreases to 426, with the probability of the most frequent solution, “build only the Red-Dead Water Conveyance and desalination plant,” increasing to 8%.

Again using the binary transformation of the flow variables f_{ij} (1= there is flow on the

arc; 0 = the arc is not used), the probability that each new connection is added in an optimal solution is seen in Table 13. The new arcs “Int 7 to Int 6,” “Int 8 to Int 7,” and “Int 8 to Int 11” all represent changes in the current direction of flow in the NWC (from north to south) to south to north, and in the case of arc “Int 8 to Int 11,” a connection that moves water from to west to east. In addition to the arcs listed in Table 13, the following new arcs were included 100% of the time: Amman to AzZarqa, AzZarqa to Irbid, and the NWC to Gaza.

Table 13: Probability that a new arc is included in the least cost solution

| | | | |
|-----------------------|-----|---------------------|-------|
| Med-Dead to Jerusalem | 56% | Beersheba to NWC | 22% |
| Med-Dead to Amman | 56% | Jerusalem to Hebron | 0.2% |
| Med-Dead to Beersheba | 33% | Jericho to Nablus | 10% |
| Med-Dead to Jericho | 40% | Nablus to Jericho | 29% |
| Med-Dead to Hebron | 56% | NWC to Nablus | 98% |
| Red-Dead to Jerusalem | 44% | Int 7 to Int 6 | 3% |
| Red-Dead to Amman | 44% | Int 8 to Int 7 | 0.6% |
| Red-Dead to Beersheba | 44% | Int 8 to Int 11 | 99.9% |
| Red-Dead to Hebron | 44% | | |

By looking at the unconverted f_{ij} values (the actual flow needed to satisfy demand) the probability distributions for the capacity needed on each arc can also be calculated. For example, the results show that the Dead Sea desalination plant to Jerusalem connection is in 44% of the optimal solutions, and when this connection is made, the capacity needed on this arc follows a distribution as show in Figure 14.

Conclusions

Bayesian Mixed Integer Programming as a new way to incorporate uncertainty into optimization models. In this method a Bayesian framework is used to construct posterior predictive distribution of input parameters, sample them, and then use the samples as parameter inputs

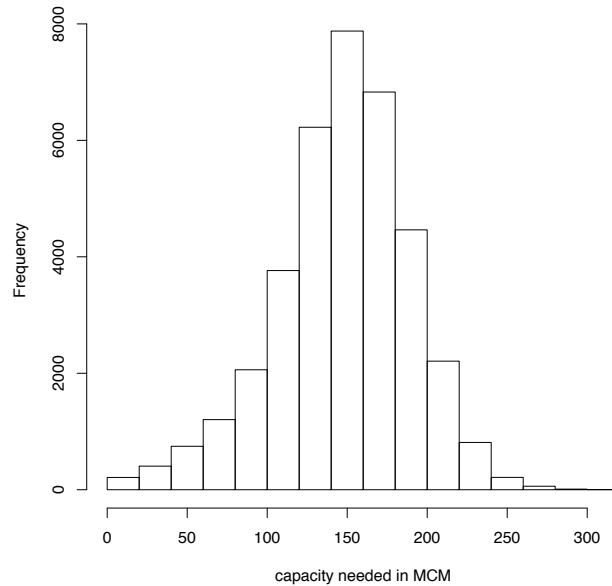


Figure 14: Histogram of the capacity needed on the connection between the Red-Dead desalination plant and Jerusalem

into the MIP. The approach facilitates wide ranging probabilistic analyses of solutions and the nature of individual decisions that comprise these solutions.

This framework is applied to the problem of modifying a water distribution network of Israel, the Palestinian Territories, and Jordan. The deterministic model indicates that the best option for the region is to install the Med-Dead Water Conveyance, but the BMIP process finds that the most probable solution includes building the Red-Dead Water Conveyance, with a probability of 8%.

While this research attempted to incorporate some of the uncertainty involved in optimizing the Middle East water distribution network, an additional extension might involve modeling the uncertain effects of climate change on the existing ground water and surface water supply. Further research might also include life-cycle analysis of component installation and operational costs, quantification of the political and social costs, and quantification

of other environmental impacts such as the environmental impact of the brine discharge and the risk of salt-water pipes breaking.

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Appendix

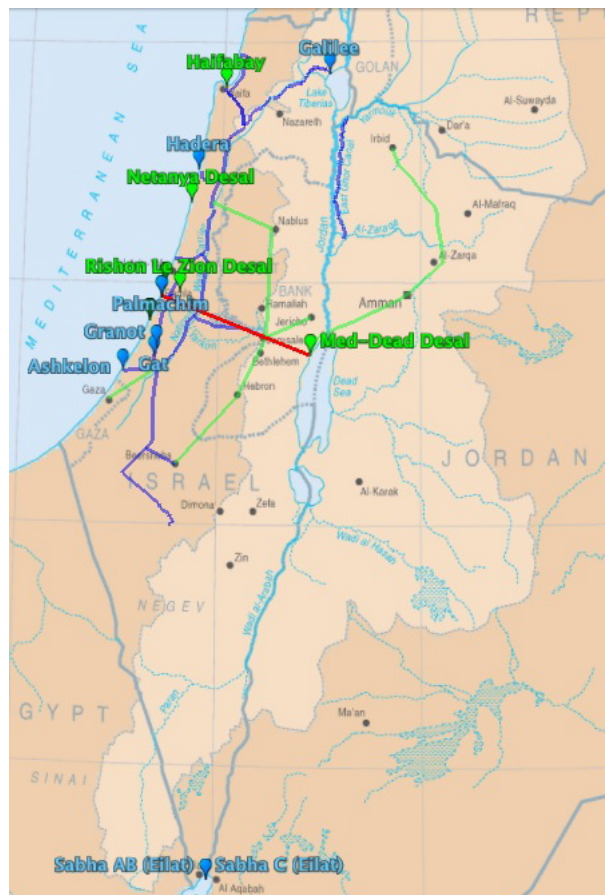


Figure 15: Solution to the model where cost is capped at \$1.7B



Figure 16: Solution to the model where cost is capped at \$1.6B



Figure 17: Solution to the model where cost is capped at \$1.5B



Figure 18: Solution to the model where cost is capped at \$1.4B

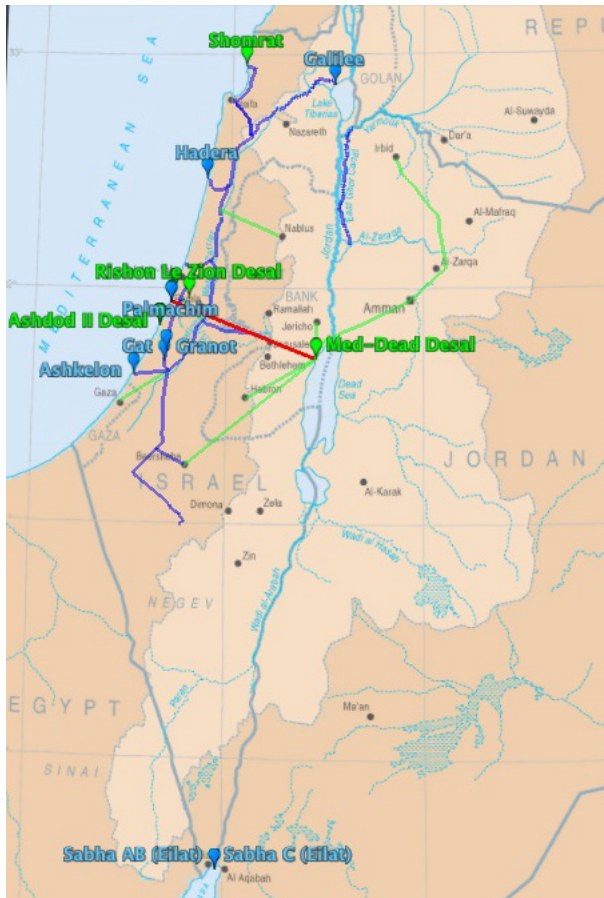


Figure 19: Solution to the model where cost is capped at \$1.3B

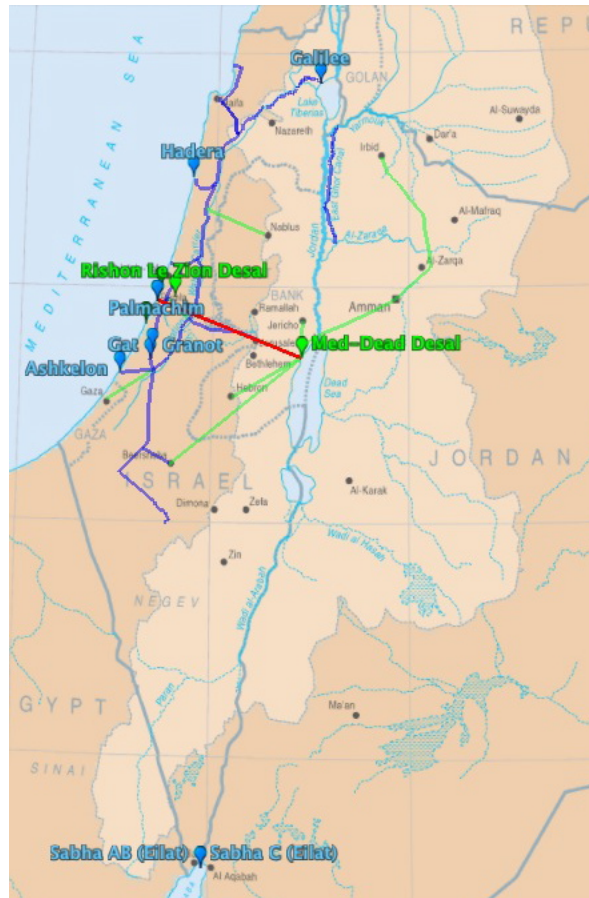


Figure 20: Solution to the model where cost is capped at \$1.2B

Table 14: Population figures from World Bank

| Year | Israel | Pal. Terr. | Jordan |
|------|-----------|------------|-----------|
| 1990 | 4,660,000 | 1,969,967 | 3,170,000 |
| 1991 | 4,949,000 | 2,047,783 | 3,545,000 |
| 1992 | 5,123,000 | 2,129,780 | 3,733,000 |
| 1993 | 5,261,000 | 2,215,059 | 3,905,931 |
| 1994 | 5,399,000 | 2,303,754 | 4,060,840 |
| 1995 | 5,545,000 | 2,396,000 | 4,195,000 |
| 1996 | 5,692,000 | 2,518,000 | 4,325,045 |
| 1997 | 5,836,000 | 2,628,000 | 4,459,212 |
| 1998 | 5,971,000 | 2,720,055 | 4,597,400 |
| 1999 | 6,125,000 | 2,815,334 | 4,680,500 |
| 2000 | 6,289,000 | 2,913,950 | 4,797,500 |
| 2001 | 6,439,000 | 3,016,021 | 4,917,500 |
| 2002 | 6,570,000 | 3,121,667 | 5,038,000 |
| 2003 | 6,689,700 | 3,231,014 | 5,164,000 |
| 2004 | 6,809,000 | 3,344,191 | 5,290,000 |
| 2005 | 6,930,100 | 3,461,333 | 5,411,500 |
| 2006 | 7,053,700 | 3,582,557 | 5,537,600 |
| 2007 | 7,180,100 | 3,708,069 | 5,718,855 |
| 2008 | 7,308,100 | 3,837,957 | 5,906,043 |

Table 15: Israeli Population by District

| District | Model Node | Population | Percent of Population |
|--------------------------|-------------------|-------------------|------------------------------|
| <i>Northern</i> | | <i>1,242,100</i> | <i>17%</i> |
| | Akko | | 8% |
| | Nazareth | | 7% |
| | Golan | | 2% |
| <i>Haifa</i> | | <i>880,000</i> | <i>12%</i> |
| | Haifa | | 9% |
| | Hadera | | 3% |
| <i>Central</i> | | <i>1,770,000</i> | <i>25%</i> |
| | Netanya | | 5% |
| | Petah Tikvah | | 8% |
| | Rishon LeTsiyon | | 11% |
| <i>Jerusalem</i> | | <i>910,000</i> | <i>12%</i> |
| | Jerusalem | | 12% |
| <i>Southern</i> | | <i>1,053,000</i> | <i>14%</i> |
| | Ashdod | | 6% |
| | Beersheba | | 7% |
| | Eilat | | 1% |
| <i>Judea and Samaria</i> | | <i>290,000</i> | <i>4%</i> |
| Totals | | 7,373,000 | 100% |